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Javier Díez-Palomar, Kees Hoogland

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Javier Díez-Palomar[1] and Kees Hoogland[2]

[1]University of Barcelona, Faculty of Education, Barcelona, Spain; jdiezpalomar@ub.edu

[2]HU University of Applied Sciences Utrecht, Utrecht, The Netherlands; kees.hoogland@hu.nl

This paper explores the contributions of research to the field of adults learning mathematics (ALM) in the last twenty years. The results of the review of the literature on ALM show that the most cited studies that have been published in the last twenty years tend to focus on the field of numeracy to understand health data (such as understanding how to dose a medicine in a medical treatment). However, we know little about key aspects of how adults learn mathematics, what obstacles they encounter, and how they overcome them. This paper identifies the main gaps that ALM research faces in the coming years.

Keywords: Numeracy, Adult Education.

The concept of numeracy

In its origins, the word numeracy was closely related to the concept of quantitative literacy (Tout, 2020). Thinking quantitatively has been the focus in educational research focused on mathematics since the last century. As Sowder (1989) says, most of the research on how we learn mathematics over several decades focused on mathematical intuition, higher-order thinking, mental schemes for understanding mathematics, and the development of quantitative thinking. In the last forty years, it should be added that research in the field of mathematics didactics has diversified into other areas beyond the strictly cognitive.

However, almost all the research that has increased our understanding of how we learn mathematics and how we can develop those innate mathematical skills that we all have, for the mere fact of being human beings, focuses on early childhood, adolescence and up to the age of university studies (Carpenter, Dossey, & Koehler, 2004). Instead, in "adulthood", there are few investigations, and of those that do exist almost all related to literacy, functional literacy, dispositions towards mathematics, and mathematics embedded in different contexts (workplace, etc.). There are studies on what mathematics we use in adult life, measuring the ‘mathematics’ we know, the challenges that adults should know to be ‘mathematically competent’ in today's world and explorations into the vulnerability of certain people and certain people social groups who lack certain knowledge and skills. Nevertheless, there is a big gap (comparatively speaking) in research into how adults learn mathematics, what research has contributed to this regard, and the gaps that need to be filled in the coming years or decades.

The objective of this paper is to review what research is being carried out in the last twenty years in the field of ALM, to identify what are the topics that are being investigated, and where there are gaps that would open new lines of research in the future on how adults learn mathematics.

Background

The first piece of evidence we have about ALM is that adults already know maths. Mathematical knowledge (the ability to think and reason in mathematical terms) is an innate skill. In the same way, that speech is a human skill that manifests itself early when children begin to speak. Mathematical thinking is also part of our ability to represent the world and solve problems. Therefore, when we talk about learning mathematics as adults we talk about academic, formal and 'pencil and paper' mathematics, similar to what we refer to when we talk about literacy, for example where we speak about reading and writing in the printed letter.

Unlike literacy, where researchers claim that it refers to the knowledge and proficiency in the use of texts, in the case of numeracy, the definitions we usually use go beyond defining numeracy as the understanding and use of the 'printed number'. As Tout (2020) says numeracy not only refers to mathematical content but is a 'way of thinking, of reasoning, of acting'. It has more to do with a set of skills related to 'numerated' behavior that goes beyond the simple use of mathematical objects and their representations. That is why it is difficult to establish the boundaries of the concept and, therefore, establish the bases of didactic research on how to 'learn' numeracy. Numeracy seems to be an innate human trait. However, it is also learned and, from the point of view of contents, procedures and ways of thinking and reasoning mathematically, it is a complex set of knowledge.

Another problem is the invisibility that some authors, such as Wedege (2010), have pointed out around mathematics and numeracy. That is, there are forms of thought and reasoning that are typical of mathematics. However, as they are part of our way of representing and understanding the world around us, they are not recognized as 'mathematics' (nor as numeracy). Examples that help to understand this paradox (well-studied in the field of ALM) are, for example, making estimates, identifying quantities, and comparing them with each other, considering risks associated with the subjective perception of probability, etc. Many people play the lottery because they are hoping to win the prize. They also decide to go out for the weekend after watching the weather forecast on TV. However, it is more difficult to find people who, before buying the lottery number, use Laplace's rule to calculate the probability they have of winning the prize, we do not make a count using the laws of combinatorics of how many different chances there are of obtaining the six winning numbers of the weekly lottery (among the 36 possible), nor do we use the density distribution of the event 'being sunny' or 'raining' when we prepare to spend the weekend away from home.

This apparent 'over-definition' of the concept of numeracy poses difficulties, as evidenced by the fact that it has been a concept that has been evolving since it first began to be used in the mid-twentieth century (Hoogland et al., 2019). During all these years, the concept of numeracy has gone from being defined as the knowledge of basic arithmetic objects and procedures (knowing how to add, subtract, multiply and divide, and some of the rules and properties of basic operations) to being defined as a social practice that involves the use of numbers (and mathematics by extension) to solve problems of everyday life, make decisions, value information and act on the world around us (multifaceted concept).

Teaching numeracy to adults

Different studies show that when children enter kindergarten at 3 or 4 years old, they already have developed what we would call 'number sense' (Westwood, 2021). Children know how to make comparisons between numbers, they know how to discriminate between quantities, and they know how to count accurately. So, when they get to school, what do they learn? According to research in our field, it seems clear that what they learn is the 'academic' language of mathematics, including the written representation of numbers and their characteristics and properties. They learn to reason with numbers. They are presented with 'problems' and asked to solve them, training them to develop that innate capacity that we all have, which is to solve problems.

Furthermore, that is called 'teaching math'. According to the type of mathematical object being taught, we will discuss arithmetic, algebra, geometry, etc. Mathematical objects, as we know, are part of conceptual structures, the components of which are very precisely related. For some, mathematics is the 'language of precision', such as Ernest (2003) reflects upon when he analyzes the different philosophies of mathematics.

We have some indications of explanatory models of how learning works: for example, Dreyfus and Dreyfus (2005) argue that teaching first goes through verbal descriptions and rules, then the second phase of association, and a third of automation; they affirm that learning is a more global process (holistic), in which holistic patterns are recognized because of interaction with the environment in multiple different situations. Hatano (1988) confirms this, claiming that learning goes beyond rule-based knowledge.

Nevertheless, what happens when we talk about teaching and learning mathematics in adults? There is no accumulated knowledge base on how adults learn mathematics comparable to the work that has been done with children and young people.

Therefore, in this paper, we have conducted a systematic review of the literature, looking for everything that has been written about adults and numeracy in the last twenty years to see to what extent the previous statement is true (or not).

Methods

To answer the research question posed here, we have conducted a systematic review of studies into numeracy and adults published in journals included in the Web of Science database from January 1, 2000, to June 1, 2021. To perform the query, the words *numeracy* AND *adults* were used.

The search generated a first database of a total of 843 articles published in that period. To select a sample of them, the following procedure was followed:

(1) We sorted all articles by the total number of citations, from the one with the most citations (1,836 citations) to those with no citations.

(2) The articles were grouped by quartiles, generating four groups ordered according to citations: A, B, C and D, as shown in the attached Table.

Table 1: Grouping of articles according to quartile by the level of citations

	Range of citations		Range of citations
A (25%)	19- 1,836	C (75%)	2-6
B (50%)	7-18	D (100%)	0-1

(3) From each quartile, (the first) ten articles that met the following criteria were chosen:

- Articles with a defined research question, a clearly explained methodology (that meets the criteria of replicability, reliability, and validity), a section of results, and conclusions (answer to the research question). We excluded all articles of a conceptual, reflective nature and those which are not based on empirical research.
- Articles that deal with the learning of mathematics of adults.
- Articles that comply with the dimensions and components of empirical studies from an adaptation by Taylor et al. (2021), as shown in Table 2.

Table 2. Criteria characterizing the components used to select the empirical studies on numeracy by dimension (adapted from Taylor et al. 2021)

Criteria	Dimension	Component
Clear identified research question(s)	Research Design	Rigor
Appropriate unit of study (school, classroom, adult learner,...)	Research Design	Rigor
Clear description of the research design	Research Design	Rigor
Target population defined	Research Design	Rigor
Appropriate statistical analysis	Research Design	Rigor
Appropriate qualitative analysis	Research Design	Rigor
Clear defined unit of analysis	Research Design	Rigor
Consent forms	Research Design	Rigor
Valid and reliable measurement	Research Design	Measurement validity
Size and significance effects are reported	Research Design	Reliability
Limitations are reported	Research Design	Reliability
Use of randomness as criteria for selecting the sample	Research Design	Reliability
Triangulation	Research Design	Reliability

Evidence of transferability to other contexts	Research Design	Generability
Adequate description of the sample /intended population	Research Design	Generability
Description of the research activities conducted	Research Design	Replicability
Clear information about the financial sources (grants, etc.)	Research Design	Independence of the researcher
Consistent, statistically significant positive effect on an outcome	Effectiveness	Overall effectiveness
Findings are reported across different contexts	Effectiveness	Consistency of effects
Contributions persist over time	Effectiveness	Sustainability
Clear criteria to identify the target population	Impact	Inclusion criteria
Detailed documentation of all components of the study (how, to whom, and by whom is intended to be conducted)	Impact	Implementation feasibility
A clear description of the intended effects in the consent forms	Impact	Implementation feasibility

The total sample of selected articles has been 40 articles, ten from each of the quartiles.

Results

Below are the results obtained from the analysis of the sample of forty selected articles.

First quartile

Almost all the articles in the group that generate the most citations are articles in the field of health, published in journals of medicine, nursing, or health psychology. The most recurrent theme that appears in these articles is the study of the extent to which adults who participate in them can read and interpret the instructions of the medicines to apply the doses correctly. Many studies investigate the skills of nurses to apply the doses of medicines to patients correctly. There are also some studies on the understanding of risk and probability in decision-making. From the point of view of higher-order skills, most of the selected articles focus on the processing information skill or the problem-solving skill. In half of them, we have identified the skill of critical thinking' none refer to managing situations. Regarding the contents that appear in the selected articles, most focus on quantity and number'. The two other contents that appear are 'pattern, relationships and change' and data and chance. There is no article that talks about the use of the calculator, the use of apps or spreadsheets, or digital skills.

Second quartile

In the second quartile, we find the same trend as in the first quartile: most of the articles that appear are published in health journals (medical research, neurological sciences, neuropsychiatry), although

we also find a journal of gerontology and another on literacy and numeracy studies. Finally, there are two articles published in a generic journal as Plos One. Regarding the research topics, the topics are more diverse than in the case of the investigations of the first quartile. In the previous group, we also found some studies on misunderstandings in adults with cognitive impairment when reading medication instructions. However, in addition to that topic, we find other research topics: the understanding of risk estimates using an online risk calculator when outcomes are expressed using integers, the impact of age on non-optimal decision-making, the management of domestic aspects using the Internet (shopping and banking skills), the financial literacy of adults, among others. On higher-order skills, unlike the studies reported in the first group, here we find that most refer to critical thinking or processing information. We also find somewhere that the problem-solving skills of adults are studied. From a content standpoint, most focus on quantity and number. We also find other topics more punctually, such as data and chance or the patterns, relationships, and change. It highlights that several of them include the use of digital applications.

Third quartile

In this third group of articles, some study of the field of medicine continues to appear. Nevertheless, unlike previous groups, these types of studies are the minority. The studies in this group are published in journals from various disciplines, such as education and development, policies, and assessment. As you can see, most are journals in the field of education. The topics studied are numeracy skills and labour market outcomes among indigenous populations, comparison between groups of adults from different socio-economic classes based on PIAAC results, assessment of objective and subjective health about numerical skills, perception of the incidence of specific disease and the survival rate, interactions between adults and children in contexts of learning informal mathematics (grocery store), etc. From the point of view of higher-order skills, in this group, the studies focus on skills such as processing information, problem-solving or critical thinking in the vast majority. We found no case of mathematizing or managing situations. Regarding the analyzed mathematical contents, those related to the dimension of quantity and number appear above all. In some cases, also what refers to data (using and reading statistics) and chance (making predictions, assessing the opportunity/risk of making a particular decision).

Fourth quartile

In this fourth group, the articles that are most cited are published in education or lifelong learning journals, specifically aimed at adult education. There are also some articles in specialized journals of mathematics education. In a minority way, journals from the field of health appear. From the point of view of the topics studied in the articles of this group, there are studies on how adults solve problems in the context of everyday life (evidence problems), the result of tests to measure the level of numeracy measured with tests (both standardized, such as PIAAC, and tests created *ad hoc*). There are also several studies related to motivation (emotional dimension), anxiety about mathematics, etc. Finally, we also find studies on the type of mathematics and mathematical skills needed to develop certain occupations in the labour market, mainly of an ethnographic (observational) nature. From the point of view of higher-order skills, in these studies, it is customary to study the skills of analyzing situations, processing information, and to a lesser extent, problem-solving and critical thinking. As

far as mathematical content is concerned, they usually focus on aspects of quantity and number, or data and chance.

Discussion and conclusions

The first result we have is that the vast majority of empirical studies are in the field of health and refer to aspects such as literacy and numeracy to understand the instructions regarding the dosage of medicines, or the understanding and application of medical treatments, or related to considering the pros and cons to make a particular health decision. In all these cases, the most outstanding aspect that appears in the studies is how adults can understand quantitative data, read the representations of that data (graphs, tables; but also, data of absolute frequencies, or percentages), and compare one set of data with another, to make decisions based on that understanding. In addition, study aspects include the understanding and management of risks and the making of decisions in environments of uncertainty with limited information.

A second result that we have found is that there is an excess of articles which consists of reflections, narratives, conceptualizations, etc., with as main content considerations, theoretical points of view, which are illustrated with concrete examples, discussions on research already published (meta-research), and specific cases of the knowledge of the author(s).

Third, the empirical studies on ALM suggest that adults learn from making connections: new mathematical objects are incorporated into our previous cognitive schemes. Several studies suggest that adult math learning is contextual: we use our experience knowledge (in the workplace, at home, in the supermarket, etc.) to incorporate new mathematical objects into our mathematical knowledge. To do this, we usually use skills such as processing information, problem-solving, and critical thinking.

Fourth, this literature review shows that one of the lines of research in ALM is on emotions. There are several studies related to the emotional response that adults give to mathematics and its learning.

Finally, fifth, the most remarkable result we have found is that the actual process of learning mathematics by adults is hardly investigated. This is reflected in the topics that appear in scientific databases such as WoS (Web of Science) which focuses mainly on what topics adults should learn, such as functions, logarithms, derivatives, geometric theorems, spatial thinking, probabilistic reasoning and Markov chains. We didn't find much research on what actually happens when an adult is learning a mathematical concept and how s/he solves mathematical problems (Contextually? Drawing on everyday experience? Through interactions?). We didn't find evidence if what we know from studies from different perspectives, such as the semantic fields, the cognitivist approach, socio-cultural studies, also applies to the case of adults.

Our main conclusion is that much more research is needed in the field of ALM, at least to contrast with what we know about research on learning mathematics with children and young people. Does it apply in the same way to adults? Or does adult learning works differently and, therefore, do we have to teach/support it in other ways (impact on the curriculum), and evaluate it also in other ways (impact on assessment, such as PIAAC, for example)?

Limitations

The scientific literature review study presented here is not exhaustive. A systematic literature review process has been followed, using WoS (Web of Science) to identify scientific articles on numeracy in the field of adults learning mathematics. However, we are aware that despite this effort to systematize, it is impossible to have reached the total number of studies carried out in this area and on this subject throughout the world.

The sample has a noticeable bias towards certain topics, because the different fields of research have different traditions from the point of view of citations. Thus, for example, in medicine, there is a greater tradition of citation than in other areas, which explains the over-representation in our sample of articles published in this type of journal. For further studies, it is planned to correct for this effect. But on the other hand, we must also consider that the level of citation is also an indicator of the visibility of the work.

Acknowledgements

This paper used data from Erasmus+ project *Common European Numeracy Framework (CENF)*, funded by the European Commission (2018-1-NL01-KA204-038913).

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